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The effects of an extra $U(1)$ axial condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature

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Abstract

Supported by recent lattice results, we consider a scenario in which a $U(1)$–breaking condensate survives across the chiral transition in QCD. This scenario has important consequences on the pseudoscalar–meson sector, which can be studied using an effective Lagrangian model. In particular, generalizing the results obtained in a previous paper (where the zero–temperature case was considered), we study the effects of this $U(1)$ chiral condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature.

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1. Introduction

There are evidences from some lattice results [1, 2, 3] that a new $U(1)$–breaking condensate survives across the chiral transition at T_{ch}, staying different from zero up to a temperature $T_{U(1)} > T_{ch}$. $T_{U(1)}$ is, therefore, the temperature at which the $U(1)$ axial symmetry is (effectively) restored, meaning that, for $T > T_{U(1)}$, there are no $U(1)$–breaking condensates. This scenario has important consequences on the pseudoscalar–meson sector, which can be studied using an effective Lagrangian model [4, 5, 6, 7], including also the new $U(1)$ chiral condensate. This one has the form $C_{U(1)} = \langle \mathcal{O}_{U(1)} \rangle$, where, for a theory with L light quark flavours, $\mathcal{O}_{U(1)}$ is a $2L$–fermion local operator that has the chiral transformation properties of [8]:*

$$\mathcal{O}_{U(1)} \sim \det_{st}(\bar{q}_{sR} q_{tL}) + \det_{st}(\bar{q}_{sL} q_{tR}), \tag{1.1}$$

where $s, t = 1, \dots, L$ are flavour indices; the colour indices [not explicitly indicated in Eq. (1.1)] are arranged in such a way that: *i)* $\mathcal{O}_{U(1)}$ is a colour singlet, and *ii)* $C_{U(1)} = \langle \mathcal{O}_{U(1)} \rangle$ is a *genuine* $2L$–fermion condensate, i.e., it has no *disconnected* part proportional to some power of the quark–antiquark chiral condensate $\langle \bar{q} q \rangle$ (see Refs. [6, 7, 9]).

The low–energy dynamics of the pseudoscalar mesons, including the effects due to the anomaly, the $q\bar{q}$ chiral condensate and the new $U(1)$ chiral condensate, can be described, in the limit of large number N_c of colours, and expanding to the first order in the light quark masses, by an effective Lagrangian written in terms of the topological charge density Q, the mesonic field $U_{ij} \sim \bar{q}_{jR} q_{iL}$ (up to a multiplicative constant) and the new field variable $X \sim \det(\bar{q}_{sR} q_{tL})$ (up to a multiplicative constant), associated with the new $U(1)$ condensate [4, 5, 6, 7]:

$$\begin{aligned} \mathcal{L}(U, U^\dagger, X, X^\dagger, Q) &= \frac{1}{2}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{2}\partial_\mu X \partial^\mu X^\dagger \\ &- V(U, U^\dagger, X, X^\dagger) + \frac{i}{2}\omega_1 Q \mathrm{Tr}(\ln U - \ln U^\dagger) \\ &+ \frac{i}{2}(1-\omega_1) Q (\ln X - \ln X^\dagger) + \frac{1}{2A} Q^2, \end{aligned} \tag{1.2}$$

where the potential term $V(U, U^\dagger, X, X^\dagger)$ has the form:

$$V(U, U^\dagger, X, X^\dagger) = \frac{\lambda_\pi^2}{4}\mathrm{Tr}[(U^\dagger U - \rho_\pi \mathbf{I})^2] + \frac{\lambda_X^2}{4}(X^\dagger X - \rho_X)^2$$

*Throughout this paper we use the following notations for the left–handed and right–handed quark fields: $q_{L,R} \equiv \frac{1}{2}(1 \pm \gamma_5) q$, with $\gamma_5 \equiv -i\gamma^0\gamma^1\gamma^2\gamma^3$.

$$-\frac{B_m}{2\sqrt{2}}\mathrm{Tr}(MU+M^\dagger U^\dagger)-\frac{c_1}{2\sqrt{2}}[\det(U)X^\dagger+\det(U^\dagger)X]. \quad (1.3)$$

$M=\mathrm{diag}(m_1,\ldots,m_L)$ is the quark mass matrix and A is the topological susceptibility in the pure–YM theory. (This Lagrangian generalizes the one originally proposed in Refs. [10], which included only the effects due to the anomaly and the $q\bar{q}$ chiral condensate.) All the parameters appearing in the Lagrangian must be considered as functions of the physical temperature T. In particular, the parameters ρ_π and ρ_X determine the expectation values $\langle U\rangle$ and $\langle X\rangle$ and so they are responsible for the behaviour of the theory respectively across the $SU(L)\otimes SU(L)$ and the $U(1)$ chiral phase transitions, as follows:

$$\begin{aligned}\rho_\pi|_{T<T_{ch}} &\equiv \frac{1}{2}F_\pi^2>0, \quad \rho_\pi|_{T>T_{ch}}<0;\\ \rho_X|_{T<T_{U(1)}} &\equiv \frac{1}{2}F_X^2>0, \quad \rho_X|_{T>T_{U(1)}}<0.\end{aligned} \quad (1.4)$$

The parameter F_π is the well–known pion decay constant, while the parameter F_X is related to the new $U(1)$ axial condensate. Indeed, from Eq. (1.4), $\rho_X=\frac{1}{2}F_X^2>0$ for $T<T_{U(1)}$, and therefore, from Eq. (1.3), $\langle X\rangle=F_X/\sqrt{2}\neq 0$. Remembering that $X\sim\det(\bar{q}_{sR}q_{tL})$, up to a multiplicative constant, we find that F_X is proportional to the new $2L$–fermion condensate $C_{U(1)}=\langle\mathcal{O}_{U(1)}\rangle$ introduced above.
In the same way, the pion decay constant F_π, which controls the breaking of the $SU(L)\otimes SU(L)$ symmetry, is related to the $q\bar{q}$ chiral condensate by a simple and well–known proportionality relation (see Refs. [4, 7] and references therein): $\langle\bar{q}_iq_i\rangle_{T<T_{ch}}\simeq-\frac{1}{2}B_mF_\pi$. (Moreover, in the simple case of L light quarks with the same mass m, $m_{NS}^2=mB_m/F_\pi$ is the squared mass of the non–singlet pseudoscalar mesons and one gets the well–known Gell-Mann–Oakes–Renner relation: $m_{NS}^2F_\pi^2\simeq-2m\langle\bar{q}_iq_i\rangle_{T<T_{ch}}$.)
It is not possible to find, in a simple way, the analogous relation between F_X and the new condensate $C_{U(1)}=\langle\mathcal{O}_{U(1)}\rangle$.

However, as we have shown in a previous paper [11], information on the quantity F_X (i.e., on the new $U(1)$ chiral condensate, to which it is related) can be derived, in the realistic case of $L=3$ light quarks with non–zero masses m_u, m_d and m_s, from the study of the radiative decays of the pseudoscalar mesons η and η' in two photons. In Ref. [11] only the zero–temperature case ($T=0$) has been considered and a first comparison of our results with the experimental data has been performed: the results are encouraging, pointing towards a certain evidence of a non–zero $U(1)$ axial condensate.

In this paper, generalizing the results obtained in Ref. [11], we study the effects of the $U(1)$ chiral condensate on the radiative decay $\eta'\to\gamma\gamma$ at finite temperature ($T\neq 0$), so

opening the possibility of a comparison with future heavy–ion experiments. In Section 2 we first re–discuss the radiative decays of the pseudoscalar mesons at $T = 0$, considering a *more general* electromagnetic anomaly interaction term, obtained by adding a *new* electromagnetic interaction term to the original electromagnetic anomaly term adopted in Ref. [11] [see Eqs. (2.8)–(2.10) below]. As we shall see, the inclusion of this *new* electromagnetic interaction term does not modify, for $T = 0$ (or, more generally, for $T < T_{ch}$) the decay amplitudes for the processes $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$: therefore, all the results (both analytical and numerical) obtained in Ref. [11], concerning these processes, remains unaffected. However, the *new* electromagnetic interaction term will prove to be crucial in the discussion of the $\eta' \to \gamma\gamma$ radiative decay at finite temperature (in particular for $T > T_{ch}$), which will be studied in detail in Section 3.

2. Radiative decays of the pseudoscalar mesons at $T = 0$

In order to study the radiative decays of the pseudoscalar mesons in two photons, we have to introduce the electromagnetic interaction in our effective model (1.2). Under *local* $U(1)$ electromagnetic transformations:

$$q \to q' = e^{i\theta e \mathbf{Q}} q, \quad A_\mu \to A'_\mu = A_\mu - \partial_\mu \theta, \tag{2.1}$$

the fields U and X transform as follows:

$$U \to U' = e^{i\theta e \mathbf{Q}} U e^{-i\theta e \mathbf{Q}}, \quad X \to X' = X. \tag{2.2}$$

Therefore, we have to replace the derivative of the fields $\partial_\mu U$ and $\partial_\mu X$ with the corresponding *covariant* derivatives:

$$D_\mu U = \partial_\mu U + ieA_\mu[\mathbf{Q}, U], \quad D_\mu X = \partial_\mu X. \tag{2.3}$$

Here $\mathbf{Q}$ is the quark charge matrix (in units of e, the absolute value of the electron charge):

$$\mathbf{Q} = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}. \tag{2.4}$$

In addition, we have to reproduce the effects of the electromagnetic anomaly, whose contribution to the four–divergence of the $U(1)$ axial current $J_{5,\mu} = \bar{q}\gamma_\mu\gamma_5 q$ and of the $SU(3)$ axial currents $A^a_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\tau_a}{\sqrt{2}}q$ (the matrices τ_a, with $a = 1, \ldots, 8$, are the generators of the algebra of $SU(3)$ in the fundamental representation, with normalization: $\mathrm{Tr}(\tau_a\tau_b) = \delta_{ab}$) is given by:

$$(\partial^\mu J_{5,\mu})^{e.m.}_{anomaly} = 2\mathrm{Tr}(\mathbf{Q}^2)G, \quad (\partial^\mu A^a_\mu)^{e.m.}_{anomaly} = 2\mathrm{Tr}\left(\mathbf{Q}^2\frac{\tau_a}{\sqrt{2}}\right)G, \tag{2.5}$$

where $G \equiv \frac{e^2 N_c}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ ($F_{\mu\nu}$ being the electromagnetic field–strength tensor), thus breaking the corresponding chiral symmetries. We observe that $\mathrm{Tr}(\mathbf{Q}^2\tau_a) \neq 0$ only for $a = 3$ or $a = 8$.

We must look for an interaction term $\mathcal{L}_I$ (constructed with the chiral Lagrangian fields and the electromagnetic operator G) which, under a $U(1)$ axial transformation $q \to q' = e^{-i\alpha\gamma_5}q$, transforms as:

$$U(1)_A: \quad \mathcal{L}_I \to \mathcal{L}_I + 2\alpha\mathrm{Tr}(\mathbf{Q}^2)G, \tag{2.6}$$

while, under $SU(3)$ axial transformations of the type $q \to q' = e^{-i\beta\gamma_5\tau_a/\sqrt{2}}q$ (with $a = 3, 8$), transforms as:

$$SU(3)_A: \quad \mathcal{L}_I \to \mathcal{L}_I + 2\beta\mathrm{Tr}\left(\mathbf{Q}^2\frac{\tau_a}{\sqrt{2}}\right)G. \tag{2.7}$$

By virtue of the transformation properties of the fields U and X under a $U(3)\otimes U(3)$ chiral transformation ($q_L \to V_L q_L$, $q_R \to V_R q_R \Rightarrow U \to V_L U V_R^\dagger$ and $X \to \det(V_L)\det(V_R)^* X$, where V_L and V_R are arbitrary 3×3 unitary matrices [4, 7]), one can see that the most simple term describing the electromagnetic anomaly interaction term is the following one:

$$\mathcal{L}_I = \frac{i}{2}G\mathrm{Tr}[\mathbf{Q}^2(\ln U - \ln U^\dagger)], \tag{2.8}$$

which is exactly the one originally proposed in Ref. [12] and also adopted in Ref. [11]. However, the presence of the new meson field X allows us to construct also another electromagnetic interaction term, still proportional to the pseudoscalar operator G, but totally *invariant* under $U(3)\otimes U(3)$ chiral transformations:

$$\Delta\mathcal{L}_I = f_\Delta\frac{i}{6}G\mathrm{Tr}(\mathbf{Q}^2)\left[\ln(X\det U^\dagger) - \ln(X^\dagger\det U)\right], \tag{2.9}$$

where f_Δ is an (up–to–now) arbitrary real parameter (the coefficient 1/6 has been introduced for convenience: see Section 3). We can thus add the two expressions (2.8) and (2.9) to form a new (more general) electromagnetic anomaly interaction term $\overline{\mathcal{L}}_I$, which, of course, satisfies both the transformation properties (2.6) and (2.7), exactly as $\mathcal{L}_I$:

$$\begin{aligned} \overline{\mathcal{L}}_I = \mathcal{L}_I + \Delta\mathcal{L}_I = \frac{i}{2} G \mathrm{Tr}[\mathbf{Q}^2(\ln U - \ln U^\dagger)] \\ +f_\Delta \frac{i}{6} G \mathrm{Tr}(\mathbf{Q}^2) \left[\ln(X \det U^\dagger) - \ln(X^\dagger \det U)\right] . \end{aligned} \tag{2.10}$$

Therefore, we shall consider the following effective chiral Lagrangian, which includes the new electromagnetic interaction terms described above:

$$\begin{aligned} \mathcal{L}(U, U^\dagger, X, X^\dagger, Q, A^\mu) = \frac{1}{2}\mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{1}{2}\partial_\mu X \partial^\mu X^\dagger \\ -V(U, U^\dagger, X, X^\dagger) + \frac{i}{2}\omega_1 Q \mathrm{Tr}(\ln U - \ln U^\dagger) \\ +\frac{i}{2}(1-\omega_1) Q (\ln X - \ln X^\dagger) + \frac{1}{2A} Q^2 \\ +\overline{\mathcal{L}}_I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \end{aligned} \tag{2.11}$$

where the potential term $V(U, U^\dagger, X, X^\dagger)$ is the one written in Eq. (1.3).
The decay amplitude of the generic process "$meson \to \gamma\gamma$" is entirely due to the electromagnetic anomaly interaction term $\overline{\mathcal{L}}_I$, which can be written more explicitly in terms of the meson fields π_a ($a = 1, \ldots, 8$), S_π and S_X, defined as follows [4, 6, 7]:

$$\begin{aligned} U &= \frac{F_\pi}{\sqrt{2}} \exp\left[\frac{i\sqrt{2}}{F_\pi}\left(\sum_{a=1}^{8} \pi_a \tau_a + \frac{S_\pi}{\sqrt{3}}\mathbf{I}\right)\right], \\ X &= \frac{F_X}{\sqrt{2}} \exp\left(\frac{i\sqrt{2}}{F_X} S_X\right). \end{aligned} \tag{2.12}$$

The π_a are the self–hermitian fields describing the octet pseudoscalar mesons; S_π is the usual "quark–antiquark" $SU(3)$–singlet meson field associated with U, while S_X is the "exotic" 6–fermion meson field associated with X [4, 6, 7].
Inserting the expressions (2.12) into Eq. (2.10), one finds that:

$$\overline{\mathcal{L}}_I = -G\frac{1}{3F_\pi}\left[\pi_3 + \frac{1}{\sqrt{3}}\pi_8 + \frac{2\sqrt{2}}{\sqrt{3}}S_\pi - f_\Delta \frac{2\sqrt{2}}{3F_X}(\sqrt{3}F_X S_\pi - F_\pi S_X)\right]. \tag{2.13}$$

The fields π_3, π_8, S_π, S_X mix together, while the remaining π_a are already diagonal [6]. However, neglecting the experimentally small mass difference between the quarks *up* and *down* (i.e., neglecting the experimentally small violations of the $SU(2)$ isotopic spin), also π_3 becomes diagonal and can be identified with the physical state π^0. The fields (π_8, S_π, S_X) can be written in terms of the eigenstates (η, η', η_X) as follows:

$$\begin{pmatrix} \pi_8 \\ S_\pi \\ S_X \end{pmatrix} = \mathbf{C} \begin{pmatrix} \eta \\ \eta' \\ \eta_X \end{pmatrix}, \tag{2.14}$$

where $\mathbf{C}$ is the following 3×3 orthogonal matrix [11]:

$$\mathbf{C} = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} = \begin{pmatrix} \cos\tilde{\varphi} & -\sin\tilde{\varphi} & 0 \\ \\ \sin\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}} & \cos\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}} & \frac{\sqrt{3}F_X}{F_{\eta'}} \\ \\ \sin\tilde{\varphi}\,\frac{\sqrt{3}F_X}{F_{\eta'}} & \cos\tilde{\varphi}\,\frac{\sqrt{3}F_X}{F_{\eta'}} & -\frac{F_\pi}{F_{\eta'}} \end{pmatrix}. \tag{2.15}$$

Here $F_{\eta'}$ is defined as follows [11]:

$$F_{\eta'} \equiv \sqrt{F_\pi^2 + 3F_X^2}, \tag{2.16}$$

and can be identified with the η' decay constant in the chiral limit of zero quark masses. Moreover, $\tilde{\varphi}$ is a mixing angle, which can be related to the masses of the quarks m_u, m_d, m_s, and therefore to the masses of the octet mesons, by the following relation [11]:

$$\tan\tilde{\varphi} = \frac{F_\pi F_{\eta'}}{6\sqrt{2}A}(m_\eta^2 - m_\pi^2), \tag{2.17}$$

where: $m_\pi^2 = 2B\tilde{m}$ and $m_\eta^2 = \frac{2}{3}B(\tilde{m} + 2m_s)$, with: $B \equiv \frac{B_m}{2F_\pi}$ and $\tilde{m} \equiv \frac{m_u+m_d}{2}$.
Concerning the masses of the two singlet states, we remind that [4, 5, 6, 7] the field η' has a "light" mass, in the sense of the $N_c \to \infty$ limit, being, in the chiral limit of zero quark masses:*

$$m_{\eta'}^2 = \frac{6A}{F_{\eta'}^2} = \frac{6A}{F_\pi^2 + 3F_X^2} = \mathcal{O}(\frac{1}{N_c}). \tag{2.18}$$

*The expression for the η' mass, when including the light–quark masses, reads as follows [6]: $\left(1 + 3\frac{F_X^2}{F_\pi^2}\right) m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = \frac{6A}{F_\pi^2}$, with: $m_K^2 = B(\tilde{m} + m_s)$.

(If we put $F_X = 0$, Eq. (2.18), or the corresponding expression including the light–quark masses [6] reported in the footnote, reduces to the well–known Witten–Veneziano relation for the η' mass [13].) On the contrary, the field η_X has a sort of "heavy hadronic" mass of order $\mathcal{O}(N_c^0)$ in the large–N_c limit. Both the η' and the η_X have the same quantum numbers (spin, parity and so on), but they have a different quark content: one is mostly $S_\pi \sim i(\bar{q}_L q_R - \bar{q}_R q_L)$, while the other is mostly $S_X \sim i[\det(\bar{q}_{sL} q_{tR}) - \det(\bar{q}_{sR} q_{tL})]$, as one can see from Eqs. (2.14)–(2.15).

The interaction Lagrangian (2.13), written in terms of the physical fields π^0, η, η' and η_X, reads as follows:

$$\overline{\mathcal{L}}_I \equiv -G\frac{1}{3F_\pi}\Big(\pi^0 + a_1\,\eta + a_2\,\eta' + \overline{a}_3\,\eta_X\Big), \tag{2.19}$$

where $a_i = \frac{1}{\sqrt{3}}(\alpha_i + 2\sqrt{2}\beta_i)$ (for $i = 1, 2, 3$), so that:

$$a_1 = \sqrt{\frac{1}{3}}\Big(\cos\tilde{\varphi} + 2\sqrt{2}\sin\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}}\Big), \tag{2.20}$$

$$a_2 = \sqrt{\frac{1}{3}}\Big(2\sqrt{2}\cos\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi}\Big), \tag{2.21}$$

$$a_3 = 2\sqrt{2}\Big(\frac{F_X}{F_{\eta'}}\Big), \tag{2.22}$$

and, moreover:

$$\overline{a}_3 = a_3 + \Delta a_3, \quad \text{with} : \Delta a_3 = -f_\Delta\frac{2\sqrt{2}F_{\eta'}}{3F_X}. \tag{2.23}$$

The values of the coefficients a_1, a_2 and a_3 are exactly the same which were calculated in Ref. [11]: therefore, the inclusion of the new electromagnetic interaction term (2.9) in the expression for the electromagnetic anomaly interaction term (2.10) only modifies (for $T = 0$ or, more generally, for $T < T_{ch}$: see the discussion in the next section) the decay amplitude for the process $\eta_X \to \gamma\gamma$, while leaving unchanged the other decay amplitudes for the processes $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$. Indeed, from Eqs. (2.14) and (2.15) we derive that:

$$\eta_X = \frac{1}{F'_\eta}(\sqrt{3}F_X S_\pi - F_\pi S_X), \tag{2.24}$$

and thus we immediately see that the term proportional to f_Δ in Eq. (2.13) is simply

equal to

$$\Delta\mathcal{L}_I = -G\frac{1}{3F_\pi}\left(-f_\Delta\frac{2\sqrt{2}F_{\eta'}}{3F_X}\right)\eta_X = -G\frac{1}{3F_\pi}\Delta a_3\eta_X. \tag{2.25}$$

The expressions for the decay amplitudes are:

$$A(\pi^0 \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} I, \tag{2.26}$$

$$A(\eta \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi}\sqrt{\frac{1}{3}}\Big(\cos\tilde{\varphi} + 2\sqrt{2}\sin\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}}\Big) I, \tag{2.27}$$

$$A(\eta' \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi}\sqrt{\frac{1}{3}}\Big(2\sqrt{2}\cos\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi}\Big) I, \tag{2.28}$$

$$A(\eta_X \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} 2\sqrt{2}\Big(\frac{F_X}{F_{\eta'}} - f_\Delta\frac{F_{\eta'}}{3F_X}\Big) I, \tag{2.29}$$

where $I \equiv \varepsilon_{\mu\nu\rho\sigma} k_1^\mu \epsilon_1^{\nu *} k_2^\rho \epsilon_2^{\sigma *}$ (k_1, k_2 being the four–momenta of the two final photons and ϵ_1, ϵ_2 their polarizations). Consequently, the following decay rates (in the real case $N_c = 3$) are derived:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^3}{64\pi^3 F_\pi^2}, \tag{2.30}$$

$$\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{192\pi^3 F_\pi^2}\Big(\cos\tilde{\varphi} + 2\sqrt{2}\sin\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}}\Big)^2, \tag{2.31}$$

$$\Gamma(\eta' \to \gamma\gamma) = \frac{\alpha^2 m_{\eta'}^3}{192\pi^3 F_\pi^2}\Big(2\sqrt{2}\cos\tilde{\varphi}\,\frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi}\Big)^2, \tag{2.32}$$

$$\Gamma(\eta_X \to \gamma\gamma) = \frac{\alpha^2 m_{\eta_X}^3}{8\pi^3 F_\pi^2}\Big(\frac{F_X}{F_{\eta'}} - f_\Delta\frac{F_{\eta'}}{3F_X}\Big)^2, \tag{2.33}$$

where $\alpha = e^2/4\pi \simeq 1/137$ is the fine–structure constant.
The results (2.30)–(2.32) are exactly the same which were found in Ref. [11]. (If we put $F_X = 0$, i.e., if we neglect the new $U(1)$ chiral condensate, the expressions written above reduce to the corresponding ones derived in Ref. [12] using an effective Lagrangian which includes only the usual $q\bar{q}$ chiral condensate.) Therefore, also the numerical results obtained in Ref. [11], concerning the processes $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$, remains unaffected. In particular, using the experimental values for the various quantities which appear in Eqs. (2.31) and (2.32), i.e.,

$$F_\pi = 92.4(4)\ \text{MeV},$$

$$
\begin{aligned}
&m_\eta = 547.30(12)\ \text{MeV},\\
&m_{\eta'} = 957.78(14)\ \text{MeV},\\
&\Gamma(\eta \to \gamma\gamma) = 0.46(4)\ \text{KeV},\\
&\Gamma(\eta' \to \gamma\gamma) = 4.26(19)\ \text{KeV},
\end{aligned}
\tag{2.34}
$$

we can extract the following values for the quantity F_X and for the mixing angle $\tilde{\varphi}$ [11]:

$$
F_X = 27(9)\ \text{MeV}, \quad \tilde{\varphi} = 16(3)^0, \tag{2.35}
$$

and these values are perfectly consistent with the relation (2.17) for the mixing angle, if we use for the pure–YM topological susceptibility the estimate $A = (180 \pm 5\ \text{MeV})^4$, obtained from lattice simulations [14].

Nevertheless, the *new* electromagnetic interaction term will play a crucial role in the discussion of the $\eta' \to \gamma\gamma$ radiative decay at finite temperature, in particular for $T > T_{ch}$: this will be studied in detail in the next section.

3. Radiative decays of the pseudoscalar mesons at $T \neq 0$

We want now to address the finite–temperature case ($T \neq 0$). As already said in the Introduction, this will be done (using a sort of mean–field approximation) simply by considering all the parameters appearing in the Lagrangian as functions of the physical temperature T. In such a way, the results obtained in the previous section can be extended to the whole region of temperatures below the chiral transition ($T < T_{ch}$), provided that the T–dependence is included in all the parameters appearing in Eqs. (2.30)–(2.33).

What happens when approaching the chiral transition temperature T_{ch} from below ($T \to T_{ch}-$)? We know that $F_\pi(T) \to 0$ when $T \to T_{ch}-$. Let us consider, for simplicity, the chiral limit of zero quark masses. From Eq. (2.18) we see that $m^2_{\eta'} \to \frac{2A(T_{ch})}{F_X^2(T_{ch})}$ when $T \to T_{ch}-$ and, from Eqs. (2.14)–(2.15), we derive:

$$
\eta' = \frac{1}{F'_\eta}(F_\pi S_\pi + \sqrt{3}F_X S_X), \tag{3.1}
$$

so that $\eta' \to S_X$ when $T \to T_{ch}-$. In this same limit, the η' decay rate (2.32) tends to the value:

$$\Gamma(\eta' \to \gamma\gamma) \underset{T \to T_{ch}-}{\longrightarrow} \frac{\alpha^2 m^3_{\eta'}(T_{ch})}{72\pi^3 F^2_X(T_{ch})}. \tag{3.2}$$

What happens, instead, in the region of temperatures $T_{ch} < T < T_{U(1)}$, above the chiral phase transition (where the $SU(3) \otimes SU(3)$ chiral symmetry is restored, while the $U(1)$ chiral condensate is still present)? First of all, we observe that we have continuity in the mass spectrum of the theory through the chiral phase transition at $T = T_{ch}$. In fact, if we study the mass spectrum of the theory in the region of temperatures $T_{ch} < T < T_{U(1)}$ [4, 6, 7], we find that the singlet meson field S_X, associated with the field X in the chiral Lagrangian, according to the second Eq. (2.12) (instead, the first Eq. (2.12) is no more valid in this region of temperatures), has a squared mass given by (in the chiral limit): $m^2_{S_X} = \frac{2A}{F^2_X}$. This is nothing but the *would–be* Goldstone particle coming from the breaking of the $U(1)$ chiral symmetry, i.e., the η', which, for $T > T_{ch}$, is a sort of "exotic" matter field of the form $S_X \sim i[\det(\bar{q}_{sL} q_{tR}) - \det(\bar{q}_{sR} q_{tL})]$. Its existence could be proved perhaps in the near future by heavy–ion experiments.

And what about the η' radiative decay rate in the region of temperatures $T_{ch} < T < T_{U(1)}$? Since $\eta' = S_X$ above T_{ch}, the electromagnetic anomaly interaction term describing the process $\eta' \to \gamma\gamma$ for $T > T_{ch}$ is only the part of $\overline{\mathcal{L}}_I$, written in Eq. (2.10), which depends on the field X:

$$\Delta\mathcal{L}_{S_X\gamma\gamma} = f_\Delta \frac{i}{6} G \mathrm{Tr}(\mathbf{Q}^2)(\ln X - \ln X^\dagger) = -f_\Delta \frac{2\sqrt{2}}{9F_X} G S_X. \tag{3.3}$$

Form this equation we easily derive the following expression for the $\eta' \to \gamma\gamma$ decay amplitude above T_{ch}:

$$A(\eta' \to \gamma\gamma)|_{T>T_{ch}} = f_\Delta \frac{e^2 N_c \sqrt{2}}{18\pi^2 F_X} I, \tag{3.4}$$

and, consequently, the following expression for the $\eta' \to \gamma\gamma$ decay rate (in the real case $N_c = 3$) above T_{ch}:

$$\Gamma(\eta' \to \gamma\gamma)|_{T>T_{ch}} = f_\Delta \frac{\alpha^2 m^3_{\eta'}}{72\pi^3 F^2_X}. \tag{3.5}$$

If we require that $\Gamma(\eta' \to \gamma\gamma)$ is a continuous function of T across the chiral transition at T_{ch}, then from Eqs. (3.2) and (3.5) we obtain the following condition for f_Δ:

$$f_\Delta(T_{ch}) = 1. \tag{3.6}$$

This means that:

$$\Gamma(\eta' \to \gamma\gamma)|_{T=T_{ch}} = \frac{\alpha^2 m^3_{\eta'}(T_{ch})}{72\pi^3 F_X^2(T_{ch})}. \tag{3.7}$$

The decay rates and the masses at finite temperature could be determined in the near–future heavy–ion experiments and then Eq. (3.7) will provide an estimate for the value of F_X at $T = T_{ch}$. Viceversa, if we were able to determine the value of F_X in some other independent way (e.g., by lattice simulations: see Ref. [11]), then Eq. (3.7) would give a theoretical estimate of the ratio $\Gamma(\eta' \to \gamma\gamma)/m^3_{\eta'}$ at $T = T_{ch}$, which could be compared with the experimental results. For example, if we make the (very plausible, indeed!) assumption that the value of F_X does not change very much going from $T = 0$ up to $T = T_{ch}$ (it will vanish at a temperature $T_{U(1)}$ above T_{ch}), i.e., $F_X(T_{ch}) \simeq F_X(0)$, and if we take for $F_X(0)$ the value reported in Eq. (2.35), then Eq. (3.7) furnishes the following estimate:

$$\Gamma(\eta' \to \gamma\gamma)|_{T=T_{ch}}/m^3_{\eta'}(T_{ch}) = \frac{\alpha^2}{72\pi^3 F_X^2(T_{ch})} \simeq (3.3^{+4.1}_{-1.4}) \times 10^{-11} \text{ MeV}^{-2}. \tag{3.8}$$

In other words, comparing with the corresponding quantities at $T = 0$, reported in Eq. (2.34), one gets that:

$$\frac{\Gamma(\eta' \to \gamma\gamma)|_{T=T_{ch}}/m^3_{\eta'}(T_{ch})}{\Gamma(\eta' \to \gamma\gamma)|_{T=0}/m^3_{\eta'}(0)} \simeq 7^{+8}_{-3}. \tag{3.9}$$

Thus, even with very large errors, due to our poor knowledge of the value of F_X, there is a quite definite prediction that the ratio $\Gamma(\eta' \to \gamma\gamma)/m^3_{\eta'}$ should have a sharp increase approaching the chiral transition temperature T_{ch}. (Of course, a smaller value of F_X would result in a larger value for the ratio in Eq. (3.9), and this case seems indeed to be favoured from the upper limit $F_X \lesssim 20$ MeV obtained from the *generalized* Witten–Veneziano formula for the η' mass [6].) One could also argue that it is physically plausible that the η' mass (of the order of 1 GeV) remains practically unchanged when going from $T = 0$ up to T_{ch} (which, from lattice simulations, is known to be of the order of 170 MeV: see, e.g., Ref. [2]): in that case, Eq. (3.9) would give an estimate for the ratio between the η' decay rates at $T = T_{ch}$ and $T = 0$. However, we want to stress that our result (3.9) is more general and does not rely on any given assumption on the behaviour of $m_{\eta'}(T)$ with the temperature T.

4. Conclusions

There are evidences from some lattice results that a new $U(1)$–breaking condensate survives across the chiral transition at T_{ch}, staying different from zero up to $T_{U(1)} > T_{ch}$. This fact has important consequences on the pseudoscalar–meson sector, which can be studied using an effective Lagrangian model, including also the new $U(1)$ chiral condensate. This model could perhaps be verified in the near future by heavy–ion experiments, by analysing the pseudoscalar–meson spectrum in the singlet sector.

In Ref. [11] we have also investigated the effects of the new $U(1)$ chiral condensate on the radiative decays, at $T = 0$, of the pseudoscalar mesons η and η' in two photons. A first comparison of our results with the experimental data has been performed: the results are encouraging, pointing towards a certain evidence of a non–zero $U(1)$ axial condensate. In this paper, generalizing the results obtained in Ref. [11], we have studied the effects of the $U(1)$ chiral condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature ($T \neq 0$). In particular, we have been able to get a quite definite theoretical prediction [see Eq. (3.9)] for the ratio between the $\eta' \to \gamma\gamma$ decay rate and the third power of the η' mass in the proximity of the chiral transition temperature T_{ch} (which, from lattice simulations, is expected to be of the order of 170 MeV): this prediction could in principle be tested in future heavy–ion experiments.

However, as we have already stressed in the conclusions of Ref. [11], one should keep in mind that our results have been derived from a very simplified model, obtained doing a first–order expansion in $1/N_c$ and in the quark masses. We expect that such a model can furnish only qualitative or, at most, "semi–quantitative" predictions. When going beyond the leading order in $1/N_c$, it becomes necessary to take into account questions of renormalization–group behaviour of the various quantities and operators involved in our theoretical analysis. This issue has been widely discussed in the literature, both in relation to the proton–spin crisis problem [15], and also in relation to the study of the η, η' radiative decays [16]. Further studies are therefore necessary in order to continue this analysis from a more quantitative point of view. We expect that some progress will be done along this line in the near future.

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